Faddeev-Jackiw analysis of topological mass-generating action

1998 J. Phys. A: Math. Gen. 31 7809

(http://iopscience.iop.org/0305-4470/31/38/016)

J. Phys. A: Math. Gen. **31** (1998) 7809–7820. Printed in the UK PII: S0305-4470(98)92792-5

Faddeev–Jackiw analysis of topological mass-generating action

Chang-Yeong Lee†§ and Dong Won Lee‡‖

† Department of Physics, Sejong University, Seoul 143-747, South Korea

‡ Department of Physics, Kon-kuk University, Seoul 143-701, South Korea

Received 25 March 1998, in final form 10 July 1998

Abstract. We analyse the gauge symmetry of a topological mass-generating action in four dimensions which contains both a vector and a second-rank antisymmetric tensor fields. In the Abelian case, this system induces an effective mass for the vector gauge field via a topological coupling $B \wedge F$ in the presence of a kinetic term for the antisymmetric tensor field B, while maintaining a gauge symmetry. On the other hand, for the non-Abelian case the B field does not have a gauge symmetry unless an auxiliary vector field is introduced to the system. We analyse this change of symmetry in the Faddeev–Jackiw formalism, and show how the auxiliary vector field enhances the symmetry. At the same time this enhanced gauge symmetry becomes reducible. We also show this phenomenon in this analysis.

1. Introduction

In 1991, there appeared a proposal that a vector field with Abelian gauge symmetry in four dimensions can develop an effective mass via a topological coupling with an antisymmetric tensor field, while maintaining the symmetry [1]. For the non-Abelian case, it was then shown that an auxiliary vector field should be introduced to the system in order to have the same symmetry property as in the Abelian case, that is both the vector and antisymmetric tensor fields behave as gauge fields [2, 3]. Straightforward extension of the Abelian case to the non-Abelian one does not work; no gauge symmetry for the antisymmetric tensor field. In [2], this was shown in the geometric BRST formalism. There a clue for the understanding of this property came from the analysis of the constraints among the equations of motion in both cases. However, from the symmetry viewpoint this understanding is not quite enough.

In this paper, we analyse the symmetry property of this topological mass-generating action in the Faddeev–Jackiw formalism. The Faddeev–Jackiw formalism [4, 5] is good for analysing the symmetry structure of a constrained system in the Hamiltonian formalism when the Lagrangian is first order in time derivatives.

To understand the Faddeev–Jackiw method, we now consider a system of N bosonic degrees of freedom, described by the Lagrangian

$$L = a_k(q)\dot{q}_k - V(q) \qquad k = 1, \ldots, N. \tag{1}$$

Then, the equations of motion are given by

$$f_{ij}\dot{q}_j - \frac{\partial V}{\partial q_i} = 0 \tag{2}$$

§ E-mail address: leecy@phy.sejong.ac.kr

‖ E-mail address: theory@kkucc.konkuk.ac.kr

where the components of the symplectic two form $f(q) = \mathrm{d}a(q)$ are given by

$$f_{ij} = \frac{\partial a_j}{\partial q_i} - \frac{\partial a_i}{\partial q_j}. \tag{3}$$

Here, $a = a_i \mathrm{d}q_i$ is a canonical one form whose components are given by the coefficients of $\dot{q}_k$ in the Lagrangian (1). If the symplectic matrix given by f_{ij} is non-singular, then its inverse matrix provides the values for the Dirac brackets of the theory [6]. However, if the matrix f_{ij} is singular, then there will be constraints from the self-consistency condition of the equations of motion [7], which one can obtain by multiplying the zero modes of the singular matrix to the equations of motion (2):

$$\Omega^J \equiv (v_i^J)^T \frac{\partial V(q)}{\partial q_i} = 0 \tag{4}$$

where the zero modes satisfy

$$(v_i^J)^T f_{ij} = 0 \qquad J = 1, \ldots, M \tag{5}$$

and M is the number of independent zero modes of f_{ij}. There are two cases for consistency equations, equation (4) [8–10]. The first case is when all the equations vanish identically. This case corresponds to a theory with gauge symmetry. In this case one can simply choose a gauge and resolve the singularity. The second case is when all or some of the equations give relations between q's. These relations among q's are constraints, and one needs to change the Lagrangian into the following form to incorporate these constraints.

$$L = a_k(q)\dot{q}_k - \eta_J \Omega^J - V(q) \qquad k = 1, \ldots, N, J = 1, \ldots m, 0 < m \leqslant N \tag{6}$$

where η_J are Lagrange multipliers. The constraints should hold under time evolution and this can be incorporated by putting the following constraints [7, 9]

$$\dot{\Omega}_J = 0, J = 1, \ldots m, 0 < m \leqslant N$$

which we implement by writing the Lagragian as

$$L = a_k(q)\dot{q}_k + \Omega^J \dot{\lambda}_J - V(q) \qquad k = 1, \ldots, N, J = 1, \ldots m, 0 < m \leqslant N. \tag{7}$$

Here we have changed the Lagrange multiplier field from η_J to λ_J. Now, we have to check whether new constraints arise from this new Lagrangian by repeating the above procedure, regarding q_k, λ_J as fields this time. If the new symplectic matrix is singular we repeat the whole procedure once again: if all the consistency conditions for the equations of motion identically vanish, thus having only the gauge symmetry, then we only have to do a gauge fixing. The gauge fixing now makes the symplectic matrix non-singular. On the other hand, if new constraints for the fields q_k, λ_J arise, then we have to repeat the whole procedure once again. We have to repeat this process until the symplectic matrix becomes non-singular. The first case occurs when the theory has only first-class constraints in the Dirac formalism, and the second case occurs when the theory possesses both first-class (gauge symmetry) and second-class constraints in the Dirac formalism. In this paper, we apply this method to analyse the symmetry of the topological mass-generating action which contains both a vector and an antisymmetric tensor fields.

So far, the antisymmetric tensor gauge theory has been analysed by many in the Abelian case [11–13]. In the non-Abelian case, however, the analysis of the symmetry structure has not been done in the Hamiltonian formalism, probably due to its complicated constraint structure. The non-Abelian case was studied earlier by Freedman and Townsend [14], but so far its quantization has been carried out only in the geometric BRST formalism [15–17, 2], and we would like to analyse the symmetry structure of the invariant action used in these works.

In section 2, we analyse the symmetry of the action with no auxiliary vector field, and show that only the vector gauge field has non-Abelian symmetry. In section 3, we analyse the symmetry after incorporating a vector auxiliary field into the action, and show that both the vector and antisymmetric tensor fields have non-Abelian gauge symmetry. In this case, the symmetry becomes reducible. In section 4, we conclude with discussions.

2. Faddeev–Jackiw analysis of the action without a vector auxiliary field

We first start with the action extended from the Abelian case straightforwardly

$$\int \mathrm{d}^4x\, \mathcal{L} = \int \mathrm{d}^4x\ \mathrm{Tr}\left\{-\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m}{4}\epsilon_{\mu\nu\rho\sigma}B^{\mu\nu}F^{\rho\sigma}\right\} \tag{8}$$

where

$$H_{\mu\nu\rho} = D_{[\mu}B_{\nu\rho]} = D_\mu B_{\nu\rho} + D_\nu B_{\rho\mu} + D_\rho B_{\mu\nu} \tag{9}$$

and $D_\mu B_{\nu\rho} = \partial_\mu B_{\nu\rho} + [A_\mu, B_{\nu\rho}]$. In the constraint analysis, the effect of the presence of the $B \wedge F$ term in the action (8) is that it only adds a few terms to the constraints in such a way that it does not change the relations among constraints. That is, even if we omit the $B \wedge F$ term from the action the relations among the constraints remain the same. Thus, from now on we shall drop the $B \wedge F$ term from our analysis to make our analysis simpler. For the metric, we will use $g_{\mu\nu} = (-, +, +, +)$ throughout the paper.

Introducing the conjugate momenta

$$\begin{aligned} \Pi_{ij} &= \dot{B}_{ij} + D_i B_{j0} - D_j B_{i0} + [A_0, B_{ij}] \\ \Pi_i &= 2(\dot{A}_i - D_i A_0) \end{aligned} \tag{10}$$

we can write the above Lagrangian in terms of conjugate momenta

$$\mathcal{L} = \tfrac{1}{4}\Pi^a_{ij}\dot{B}^a_{ij} + \tfrac{1}{4}\Pi^a_i\dot{A}^a_i - V_{(0)} \tag{11}$$

where

$$V_{(0)} = \tfrac{1}{2}\Pi^a_{ij}D_j B^a_{i0} - \tfrac{1}{4}\Pi^a_{ij}[A_0, B_{ij}]^a + \tfrac{1}{8}\Pi^2_{ij} + \tfrac{1}{4}\Pi^a_i D_i A^a_0 + \tfrac{1}{16}\Pi^2_i + \tfrac{1}{8}F^2_{ij} + \tfrac{1}{24}H^2_{ijk}. \tag{12}$$

From this Lagrangian we first get the components of the canonical one form, then we calculate a symplectic matrix with symplectic variables $B^a_{0i}, B^a_{ij}, \Pi^a_{ij}, A^a_0, A^a_i$ and Π^a_i (in order of appearance in the matrix). With this symplectic matrix, we write a matrix equation for zero modes:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P & 0 & 0 & 0 \\ 0 & P' & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & T \\ 0 & 0 & 0 & 0 & T' & 0 \end{bmatrix} \begin{bmatrix} \alpha^g_l \\ \beta^g_{lm} \\ \gamma^g_{lm} \\ \rho^g_0 \\ \sigma^g_l \\ \kappa^g_l \end{bmatrix} = 0 \tag{13}$$

where

$$\begin{aligned} P &\equiv f^{(0)B\Pi}_{ija\ lmg} = -\tfrac{1}{4}\delta^{ag}\delta^{lm}_{ij}\delta(\boldsymbol{x}-\boldsymbol{y}) \\ P' &\equiv f^{(0)\Pi B}_{ija\ lmg} = \tfrac{1}{4}\delta^{ag}\delta^{lm}_{ij}\delta(\boldsymbol{x}-\boldsymbol{y}) \\ T &\equiv f^{(0)A\Pi}_{ia\ lg} = -\tfrac{1}{4}\delta_{il}\delta^{ag}\delta(\boldsymbol{x}-\boldsymbol{y}) \\ T' &\equiv f^{(0)\Pi A}_{ia\ lg} = \tfrac{1}{4}\delta_{il}\delta^{ag}\delta(\boldsymbol{x}-\boldsymbol{y}). \end{aligned}$$

Throughout this paper it will be understood that all quantities are taken at equal time. The above symplectic matrix is singular because there exist nontrivial eigenvectors with zero eigenvalue. Now we can write new constraints from the zero modes as

$$\Omega^{(0)} = \int \mathrm{d}^3\boldsymbol{x} \left\{ \alpha_l^g(\boldsymbol{x}) \frac{\delta}{\delta B_{0l}^g(\boldsymbol{x})} + \rho_0^g(\boldsymbol{x}) \frac{\delta}{\delta A_0^g(\boldsymbol{x})} \right\} \int \mathrm{d}^3\boldsymbol{y}\, V_{(0)}. \tag{14}$$

Since $\alpha_l^g(\boldsymbol{x})$ and $\rho_0^g(\boldsymbol{x})$ are arbitrary parameters, we write the constraints and their Lagrange multipliers as follows

$$\begin{aligned} \Omega_1^{(0)} &= D_j \Pi_{ji}^a, \eta_i^a \\ \Omega_2^{(0)} &= D_i \Pi_i^a + [B_{ij}, \Pi_{ij}]^a, \omega^a. \end{aligned} \tag{15}$$

Incorporating these new constraints, the Lagrangian now becomes

$$\mathcal{L} = \tfrac{1}{4}\Pi_{ij}^a \dot{B}_{ij}^a + \tfrac{1}{4}\Pi_i^a \dot{A}_i^a + (D_j\Pi_{ji})^a \dot{\eta}_i^a + (D_i\Pi_i + [B_{ij}, \Pi_{ij}])^a \dot{\omega}^a - V_{(1)} \tag{16}$$

where

$$V_{(1)} = \tfrac{1}{8}\Pi_{ij}^2 + \tfrac{1}{16}\Pi_i^2 + \tfrac{1}{8}F_{ij}^2 + \tfrac{1}{24}H_{ijk}^2. \tag{17}$$

Repeating the same procedure, we obtain a new symplectic matrix, and write a matrix equation for zero modes as follows

$$\begin{bmatrix} 0 & P & 0 & 0 & 0 & R \\ P' & 0 & 0 & 0 & S & U \\ 0 & 0 & 0 & T & V & W \\ 0 & 0 & T' & 0 & 0 & X \\ 0 & S' & V' & 0 & 0 & 0 \\ R' & U' & W' & X' & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_{lm}^g \\ \beta_{lm}^g \\ \gamma_l^g \\ \rho_l^g \\ \sigma_l^g \\ \nu^g \end{bmatrix} = 0 \tag{18}$$

where

$$\begin{aligned} R' &\equiv f^{gac}\Pi_{ij}^c \delta(\boldsymbol{x}-\boldsymbol{y}) & R' &\equiv -f^{agc}\Pi_{lm}^c \delta(\boldsymbol{x}-\boldsymbol{y}) \\ S &\equiv D_m^y \delta^{ag}\delta_{ij}^{ml} \delta(\boldsymbol{x}-\boldsymbol{y}) & S' &\equiv -D_j^x \delta^{ag}\delta_{ji}^{lm} \delta(\boldsymbol{x}-\boldsymbol{y}) \\ U &\equiv f^{gba} B_{ij}^b \delta(\boldsymbol{x}-\boldsymbol{y}) & U' &\equiv -f^{abg} B_{lm}^b \delta(\boldsymbol{x}-\boldsymbol{y}) \\ V &\equiv f^{gac}\Pi_{il}^c \delta(\boldsymbol{x}-\boldsymbol{y}) & V' &\equiv -f^{agc}\Pi_{li}^c \delta(\boldsymbol{x}-\boldsymbol{y}) \\ W &\equiv f^{gac}\Pi_i^c \delta(\boldsymbol{x}-\boldsymbol{y}) & W' &\equiv -f^{agc}\Pi_l^c \delta(\boldsymbol{x}-\boldsymbol{y}) \\ X &\equiv -D_i \delta^{ag} \delta(\boldsymbol{x}-\boldsymbol{y}) & X' &\equiv -D_l \delta^{ag} \delta(\boldsymbol{x}-\boldsymbol{y}). \end{aligned}$$

Here, symplectic variables are $B_{ij}^a, \Pi_{ij}^a, A_i^a, \Pi_i^a, \eta_i^a$ and ω^a in order of appearance in the symplectic matrix. From the above matrix equation, we find two zero modes with independent parameters σ_l and ν:

$$(\alpha_{lm} = 4[B_{lm}, \nu], \beta_{lm} = 4[\Pi_{lm}, \nu], \gamma_l = 4D_i\nu, \rho_l = 4[\Pi_i, \nu], \sigma_l, \nu). \tag{19}$$

Among these two zero modes, only the zero mode with σ_l provides a new constraint

$$\Omega_1^{(1)} = [F_{jk}, H_{ijk}] - [\Pi_j, \Pi_{ji}]. \tag{20}$$

The consistency condition from the zero mode which is related to ν vanishes identically. Thus the new Lagrangian is given by

$$\begin{aligned} \mathcal{L} = \tfrac{1}{4}\Pi_{ij}^a \dot{B}_{ij}^a &+ \tfrac{1}{4}\Pi_i^a \dot{A}_i^a + (D_j\Pi_{ji})^a \dot{\eta}_i^a + (D_i\Pi_i + [B_{ij}, \Pi_{ij}])^a \dot{\omega}^a \\ &+([F_{jk}, H_{ijk}] - [\Pi_j, \Pi_{ji}])^a \dot{\xi}_i^a - V_{(2)} \end{aligned} \tag{21}$$

where

$$V_{(2)} = \tfrac{1}{8}\Pi_{ij}^2 + \tfrac{1}{16}\Pi_i^2 + \tfrac{1}{8}F_{ij}^2 + \tfrac{1}{24}H_{ijk}^2 \mid_{\Omega_1^{(1)}} . \tag{22}$$

Here the symplectic variable ξ_i^a is added. Then, the symplectic matrix and its zero mode equation is

$$\begin{bmatrix} 0 & P & 0 & 0 & 0 & R & \Psi \\ P' & 0 & 0 & 0 & S & U & \Sigma \\ 0 & 0 & 0 & T & V & W & \phi \\ 0 & 0 & T' & 0 & 0 & X & \chi \\ 0 & S' & V' & 0 & 0 & 0 & 0 \\ R' & U' & W' & X' & 0 & 0 & 0 \\ \Psi' & \Sigma' & \phi' & \chi' & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_{lm}^g \\ \beta_{lm}^g \\ \gamma_l^g \\ \rho_l^g \\ \sigma_l^g \\ \nu^g \\ \mu_l^g \end{bmatrix} = 0 \tag{23}$$

where

$$\Psi = f^{gbc}F_{mn}^b\{(\partial_l^y\delta^{ca} + f^{cda}A_l^d)\delta_{ij}^{mn} + (\partial_m^y\delta^{ca} + f^{cda}A_m^d)\delta_{ij}^{nl} + (\partial_n^y\delta^{ca} + f^{cda}A_n^d)\delta_{ij}^{lm}\}\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$\Psi' = -f^{abc}F_{jk}^b\{(\partial_i^x\delta^{cg} + f^{cdg}A_i^d)\delta_{lm}^{jk} + (\partial_j^x\delta^{cg} + f^{cdg}A_j^d)\delta_{lm}^{ki} + (\partial_k^x\delta^{cg} + f^{cdg}A_k^d)\delta_{lm}^{ij}\}\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$\phi = f^{gbc}\{(2\partial_j^y\delta^{ba}\delta_{ik} + f^{bae}\delta_{ij}A_k^e + f^{bda}\delta_{ki}A_j^d)\delta(\boldsymbol{x}-\boldsymbol{y})\}H_{ljk}^c + f^{gbc}f^{cae}F_{jk}^b(\delta_{li}B_{jk}^e + \delta_{ji}B_{kl}^e + \delta_{ki}B_{lj}^e)\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$\phi' = -f^{abc}\{(2\partial_j^x\delta^{bg}\delta_{kl} + f^{bge}\delta_{jl}A_k^e + f^{bdg}\delta_{kl}A_j^d)\delta(\boldsymbol{x}-\boldsymbol{y})\}H_{ijk}^c - f^{abc}f^{cge}F_{jk}^b(\delta_{li}B_{jk}^e + \delta_{jl}B_{ki}^e + \delta_{kl}B_{ij}^e)\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$\Sigma = -f^{gba}\Pi_m^b\delta_{ij}^{ml}\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$\Sigma' = f^{abg}\Pi_j^b\delta_{ji}^{lm}\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$\chi = -f^{gac}\Pi_{il}^c\delta(\boldsymbol{x}\mathrm{i}-\boldsymbol{y})$$

$$\chi' = f^{agc}\Pi_{li}^c\delta(\boldsymbol{x}-\boldsymbol{y})$$

and P, P', R, etc are the same as before. Again this symplectic matrix is singular, and after solving the zero mode equation we find a zero mode:

$$(\alpha_{ij} = 4[B_{ij}, \nu], \beta_{ij} = 4[\Pi_{ij}, \nu], \gamma_i = 4D_i\nu, \rho_i = 4[\Pi_i, \nu], \sigma_i = 0, \nu, \mu_i = 0). \tag{24}$$

With this zero mode, we see that the constraint equation vanishes identically:

$$\Omega^{(2)} = \int \mathrm{d}^3\boldsymbol{x}\left\{\{4[B_{lm}, \nu]^g\frac{\delta}{\delta B_{lm}^g} + 4[\Pi_{lm}, \nu]^g\frac{\delta}{\delta\Pi_{lm}^g} + 4[\Pi_l, \nu]^g\frac{\delta}{\delta\Pi_l^g} + 4D_l\nu^g\frac{\delta}{\delta A_l^g}\right\}\int V_{(1)}\,\mathrm{d}^3\boldsymbol{y} \equiv 0.$$

This shows that the theory we are considering has gauge symmetry and the gauge transformation is given by the above zero mode. Namely, the gauge transformations of the fields are given by $\delta B_{ij} = \alpha_{ij} = 4[B_{ij}, \nu]$, $\delta A_i = \gamma_i = 4D_i\nu$. This clearly shows that only the vector field has non-Abelian gauge symmetry unlike the Abelian case [1] where both the vector and antisymmetric tensor fields behave as gauge fields.

Finally, to remove the singularity due to the above gauge symmetry, we choose a gauge as

$$\partial_i A_i = 0. \tag{25}$$

Then the Lagrangian becomes

$$\mathcal{L} = \tfrac{1}{4}\Pi^a_{ij}\dot{B}^a_{ij} + \tfrac{1}{4}\Pi^a_i\dot{A}^a_i + (D_j\Pi_{ji})^a\dot{\eta}^a_i + (D_i\Pi_i + [B_{ij},\Pi_{ij}])^a\dot{\omega}^a$$
$$+([F_{jk},H_{ijk}] - [\Pi_j,\Pi_{ji}])^a\dot{\xi}^a_i + (\partial_i A^a_i)\dot{\lambda}^a - V_{(3)} \quad (26)$$

where

$$V_{(3)} = V_{(2)}\,|_{\partial_i A_i = 0}\ .$$

Now, the symplectic matrix with an added symplectic variable λ^a is given by

$$\begin{bmatrix} 0 & P & 0 & 0 & 0 & R & \Psi & 0 \\ P' & 0 & 0 & 0 & S & U & \Sigma & 0 \\ 0 & 0 & 0 & T & V & W & \phi & Y \\ 0 & 0 & T' & 0 & 0 & X & \chi & 0 \\ 0 & S' & V' & 0 & 0 & 0 & 0 & 0 \\ R' & U' & W' & X' & 0 & 0 & 0 & 0 \\ \Psi' & \Sigma' & \phi' & \chi' & 0 & 0 & 0 & 0 \\ 0 & 0 & Y' & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (27)$$

where

$$Y = -\partial_i\delta(\boldsymbol{x}-\boldsymbol{y})\delta^{ag} \qquad Y' = -\partial_l\delta(\boldsymbol{x}-\boldsymbol{y})\delta^{ag}$$

and P, P', R, etc are the same as before. One can check that this symplectic matrix is nonsingular, as it should be.

3. Faddeev–Jackiw analysis of the action with a vector auxiliary field

In the previous section, we have seen that the straightforward extension of the Abelian action to the non-Abelian one does not work. Thus, following [15, 16, 2], we introduce an auxiliary vector field to the theory by replacing $B_{\mu\nu} \longrightarrow B_{\mu\nu} - D_{[\mu}K_{\nu]}$, where K_ν is an auxiliary vector field. This replacement also changes the field strength of the antisymmetric tensor field into

$$H_{\mu\nu\rho} = D_{[\mu}B_{\nu\rho]} \longrightarrow H'_{\mu\nu\rho} = D_{[\mu}B_{\nu\rho]} - [F_{[\mu\nu}, K_{\rho]}]. \quad (28)$$

We now write the Lagrangian with this new field strength H'

$$\mathcal{L} = \mathrm{Tr}\{-\tfrac{1}{12}H'_{\mu\nu\rho}H'^{\mu\nu\rho} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\}. \quad (29)$$

Introducing the canonical momenta

$$\Pi_{ij} = \tfrac{1}{2}(\dot{B}_{ij} + D_iB_{j0} - D_jB_{i0} + [A_0,B_{ij}] - [\dot{A}_i,K_j] + [\dot{A}_j,K_i] + [D_iA_0,K_j]$$
$$-[D_jA_0,K_i] - [F_{ij},K_0]) \quad (30)$$
$$\Pi_i = \dot{A}_i - D_iA_0$$

we rewrite the Lagrangian in its first-order form

$$\mathcal{L} = \tfrac{1}{2}\Pi^a_{ij}\dot{B}^a_{ij} + \tfrac{1}{2}([A_i,K_j] - [A_j,K_i])^a\dot{\Pi}^a_{ij} + [A_j,\Pi_{ij}]^a\dot{K}^a_i + \tfrac{1}{2}\Pi^a_i\dot{A}^a_i - V_{(0)} \quad (31)$$

where

$$V_{(0)} = \Pi^a_{ij}D_jB^a_{i0} - \tfrac{1}{2}\Pi^a_{ij}[A_0,B_{ij}]^a + \Pi^a_{ij}[D_jA_0,K_i] + \tfrac{1}{2}\Pi^a_{ij}[F_{ij},K_0]^a + \tfrac{1}{2}\Pi^2_{ij}$$
$$+\tfrac{1}{2}\Pi^a_iD_iA^a_0 + \tfrac{1}{4}\Pi^2_i + \tfrac{1}{8}F^2_{ij} + \tfrac{1}{24}H^2_{ijk}.$$

By repeating the procedure from the previous section, we first obtain a zero mode equation for the symplectic matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P & 0 & 0 & 0 & 0 & 0 \\ 0 & P' & 0 & 0 & Q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q' & 0 & 0 & S & 0 & T \\ 0 & 0 & 0 & 0 & S' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T' & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_l^g \\ \beta_{lm}^g \\ \gamma_{lm}^g \\ \rho_0^g \\ \sigma_l^g \\ \phi_l^g \\ \mu_0^g \\ \nu_l^g \end{bmatrix} = 0 \tag{32}$$

where

$$P = -\tfrac{1}{2}\delta^{ag}\delta_{ij}^{lm}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$P' = \tfrac{1}{2}\delta^{ag}\delta_{lm}^{ij}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$Q = -\tfrac{1}{2}f^{abc}\delta^{bg}(\delta_{il}K_j^c - \delta_{jl}K_i^c)\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$Q' = \tfrac{1}{2}f^{gbc}\delta^{ba}(\delta_{il}K_m^c - \delta_{mi}K_l^c)\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$S = -\tfrac{1}{2}\delta_{il}\delta^{ag}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$S' = \tfrac{1}{2}\delta_{il}\delta^{ag}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$T = f^{gbc}\delta^{ba}\Pi_{li}^c\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$T' = -f^{abc}\delta^{bg}\Pi_{il}^c\delta(\boldsymbol{x}-\boldsymbol{y}).$$

Here the symplectic variables are $B_{0i}^a, B_{ij}^a, \Pi_{ij}^a, A_0^a, A_i^a, \Pi_i^a, K_0^a$ and K_i^a in order of appearance in the symplectic matrix. The matrix equation has four zero modes with four independent variables $\alpha_l, \rho_0, \mu_0, \nu_l$:

$$(\alpha_l, 0, 0, \rho_0, 0, 2[\Pi_{ml}, \nu_m], \mu_0, \nu_l). \tag{33}$$

These zero modes yield four constraints, and we write them with their respective Lagrange multiplier below

$$\begin{aligned} \Omega_1^{(0)} &= D_j\Pi_{ji}^a;\ \ \eta_i^a \\ \Omega_2^{(0)} &= D_i\Pi_i^a + [B_{ij}, \Pi_{ij}]^a - 2[\Pi_{ij}, D_jK_i]^a;\ \ \omega^a \\ \Omega_3^{(0)} &= [F_{ij}, \Pi_{ij}]^a;\ \ \theta^a \\ \Omega_4^{(0)} &= [\Pi_{ji}, \Pi_j]^a - \tfrac{1}{4}[H_{ijk}, F_{jk}]^a;\ \ \chi_i^a. \end{aligned} \tag{34}$$

However, these four constraints are not all independent. The first and third constraints are related by the following equation

$$D_i\Omega_1^{(0)} + \tfrac{1}{2}\Omega_3^{(0)} = 0. \tag{35}$$

Note that this reducibility condition is different from that of the Abelian case where the reducibility condition holds trivially, $\partial_i\partial_j\Pi_{ij} = 0$, due to the antisymmetry of the Π_{ij} indices [11]. Thus, in order to incorporate this dependence between the two constraints, we further introduce a new constraint and its Lagrange multiplier

$$\Omega_5^{(0)} = D_i\eta_i^a + \tfrac{1}{2}\theta^a;\ \ \lambda^a \tag{36}$$

and write the Lagrangian as

$$\begin{aligned} \mathcal{L} = {} & \tfrac{1}{2}\Pi_{ij}^a\dot{B}_{ij}^a + \tfrac{1}{2}([A_i, K_j] - [A_j, K_i])^a\dot{\Pi}_{ij}^a + [A_j, \Pi_{ij}]^a\dot{K}_i^a + \tfrac{1}{2}\Pi_i^a\dot{A}_i^a + (D_j\Pi_{ji})^a\dot{\eta}_i^a \\ & + (D_i\Pi_i + [B_{ij}, \Pi_{ij}] - 2[\Pi_{ij}, D_jK_i])^a\dot{\omega}^a \end{aligned}$$

$$+[F_{ij}, \Pi_{ij}]^a \dot{\theta}^a + ([\Pi_{ji}, \Pi_j] - \tfrac{1}{4}[H_{ijk}, F_{jk}])^a \dot{\chi}_i^a$$
$$+(D_i \eta_i^a + \tfrac{1}{2}\theta^a)\dot{\lambda}^a - V_{(1)} \qquad (37)$$

where

$$V_{(1)} = \tfrac{1}{2}\Pi_{ij}^2 + \tfrac{1}{4}\Pi_i^2 + \tfrac{1}{8}F_{ij}^2 + \tfrac{1}{8}H_{ijk}^a D_i B_{jk}^a$$
$$+\tfrac{1}{2}K_i^a[\Pi_{ij}, \Pi_j]^a. \qquad (38)$$

With the symplectic variables $B_{ij}^a, \Pi_{ij}^a, A_i^a, \Pi_i^a, K_i^a, \eta_i^a, \omega^a, \theta^a, \chi_i^a$ and λ^a (in order of appearance in the symplectic matrix), we obtain the following zero mode equation for the symplectic matrix

$$\begin{bmatrix} 0 & P & 0 & 0 & 0 & 0 & C & 0 & D & 0 \\ P' & 0 & Q & 0 & 0 & E & F & G & H & 0 \\ 0 & Q' & 0 & S & T & I & J & K & L & M \\ 0 & 0 & S' & 0 & 0 & 0 & N & 0 & A & 0 \\ 0 & 0 & T' & 0 & 0 & 0 & B & 0 & U & 0 \\ 0 & E' & I' & 0 & 0 & 0 & 0 & 0 & 0 & V \\ C' & F' & J' & N' & B' & 0 & 0 & 0 & 0 & 0 \\ 0 & G' & K' & 0 & 0 & 0 & 0 & 0 & 0 & W \\ D' & H' & L' & A' & U' & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M' & 0 & 0 & V' & 0 & W' & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_{lm}^g \\ \beta_{lm}^g \\ \gamma_l^g \\ \rho_l^g \\ \sigma_l^g \\ \mu_l^g \\ \nu^g \\ \xi^g \\ \psi_l^g \\ \phi^g \end{bmatrix} = 0 \qquad (39)$$

where

$$C = f^{gac}\Pi_{ij}^c \delta(\boldsymbol{x}-\boldsymbol{y})$$
$$C' = -f^{agc}\Pi_{lm}^c \delta(\boldsymbol{x}-\boldsymbol{y})$$
$$D = \tfrac{1}{8} f^{gbc} F_{mn}^b \{(\partial_l^y \delta^{ca} + f^{cde} A_l^d \delta^{ea})\delta_{ij}^{mn} + (\partial_m^y \delta^{ca} + f^{cde} A_m^d \delta^{ea})\delta_{ij}^{nl}$$
$$+(\partial_n^y \delta^{ca} + f^{cde} A_n^d \delta^{ea})\delta_{ij}^{lm}\}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$D' = -\tfrac{1}{8} f^{abc} F_{jk}^b \{(\partial_i^x \delta^{cg} + f^{cde} A_i^d \delta^{eg})\delta_{jk}^{lm} + (\partial_j^x \delta^{cg} + f^{cde} A_j^d \delta^{eg})\delta_{ki}^{lm}$$
$$+(\partial_k^x \delta^{cg} + f^{cde} A_k^d \delta^{eg})\delta_{ij}^{lm}\}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$E = \tfrac{1}{2}(\partial_m^y \delta^{ag} + f^{gbc} A_m^b \delta^{ca})\delta_{ij}^{ml}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$E' = -\tfrac{1}{2}(\partial_j^x \delta^{ag} + f^{abc} A_j^b \delta^{cg})\delta_{ij}^{ml}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$F = (f^{gba} B_{ij}^b + 2f^{gac}(D_i K_j^c - D_j K_i^c))\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$F' = -(f^{abg} B_{lm}^b + 2f^{agc}(D_l K_m^c - D_m K_l^c))\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$G = f^{gba} F_{ij}^b \delta(\boldsymbol{x}-\boldsymbol{y})$$
$$G' = -f^{abg} F_{lm}^b \delta(\boldsymbol{x}-\boldsymbol{y})$$
$$H = \tfrac{1}{2} f^{gac}\Pi_m^c \delta_{ij}^{ml}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$H' = -\tfrac{1}{2} f^{agc}\Pi_j^c \delta_{ji}^{lm}\delta(\boldsymbol{x}-\boldsymbol{y})$$
$$I = f^{gac}\Pi_{il}^c \delta(\boldsymbol{x}-\boldsymbol{y})$$
$$I' = -f^{agc}\Pi_{li}^c \delta(\boldsymbol{x}-\boldsymbol{y})$$
$$J = f^{gac}\Pi_i^c - 2f^{gbc} f^{cae}\Pi_{li}^b K_l^e \delta(\boldsymbol{x}-\boldsymbol{y})$$
$$J' = -f^{agc}\Pi_l^c + 2f^{abc} f^{cge}\Pi_{il} K_i^e \delta(\boldsymbol{x}-\boldsymbol{y})$$
$$K = 2f^{gbc}\{(\partial_l^y \delta^{ba} + f^{bda} A_l^d)\delta(\boldsymbol{x}-\boldsymbol{y})\}\Pi_{li}^c$$
$$K' = -2f^{abc}\{(\partial_i^x \delta^{bg} + f^{bdg} A_i^d)\delta(\boldsymbol{x}-\boldsymbol{y})\}\Pi_{il}^c$$
$$L = \tfrac{1}{2} f^{gbc}\{(\partial_j^y \delta^{ba} + f^{bda} A_j^d)\delta(\boldsymbol{x}-\boldsymbol{y})\}H_{lji}^c$$

$$
\begin{aligned}
&+\tfrac{1}{4}f^{gbc}F^b_{jk}\{f^{cae}(\delta_{il}B^e_{jk}+\delta_{ji}B^e_{kl}+\delta_{ki}B^e_{lj})\delta(\boldsymbol{x}-\boldsymbol{y})\\
&-f^{cde}((\partial^y_l\delta_{ji}\delta^{da}-\partial^y_j\delta_{li}\delta^{da}+f^{dah}\delta_{li}A^h_j+f^{dfa}\delta_{ji}A^f_l)\delta(\boldsymbol{x}-\boldsymbol{y}))K^e_k\\
&-f^{cde}((\partial^y_j\delta_{ki}\delta^{da}-\partial^y_k\delta_{ji}\delta^{da}+f^{dah}\delta_{ji}A^h_k+f^{dfa}\delta_{ki}A^f_j)\delta(\boldsymbol{x}-\boldsymbol{y}))K^e_l\\
&-f^{cde}((\partial^y_k\delta_{li}\delta^{da}-\partial^y_l\delta_{ki}\delta^{da}+f^{dah}\delta_{ki}A^h_l+f^{dfa}\delta_{li}A^f_k)\delta(\boldsymbol{x}-\boldsymbol{y}))K^e_j\}\\
L'&=-\tfrac{1}{2}f^{abc}\{(\partial^x_j\delta^{bg}+f^{bdg}A^d_j)\delta(\boldsymbol{x}-\boldsymbol{y})\}H^c_{ijl}\\
&-\tfrac{1}{4}f^{abc}F^b_{jk}\{f^{cge}(\delta_{il}B^e_{jk}+\delta_{jl}B^e_{ki}+\delta_{kl}B^e_{ij})\delta(\boldsymbol{x}-\boldsymbol{y})\\
&-f^{cde}((\partial^x_i\delta_{jl}\delta^{dg}-\partial^x_j\delta_{il}\delta^{dg}+f^{dgh}\delta_{il}A^h_j+f^{dfg}\delta_{jl}A^f_i)\delta(\boldsymbol{x}-\boldsymbol{y}))K^e_k\\
&-f^{cde}((\partial^x_j\delta_{kl}\delta^{dg}-\partial^x_k\delta_{jl}\delta^{dg}+f^{dgh}\delta_{jl}A^h_k+f^{dfg}\delta_{kl}A^f_j)\delta(\boldsymbol{x}-\boldsymbol{y}))K^e_i\\
&-f^{cde}((\partial^x_k\delta_{il}\delta^{dg}-\partial^x_i\delta_{kl}\delta^{dg}+f^{dgh}\delta_{kl}A^h_i+f^{dfg}\delta_{il}A^f_k)\delta(\boldsymbol{x}-\boldsymbol{y}))K^e_j\}\\
M&=f^{gac}\eta^c_i\delta(\boldsymbol{x}-\boldsymbol{y})\\
M'&=-f^{agc}\eta^c_l\delta(\boldsymbol{x}-\boldsymbol{y})\\
N&=-D_i\delta^{ag}\delta(\boldsymbol{x}-\boldsymbol{y})\\
N'&=-D_l\delta^{ag}\delta(\boldsymbol{x}-\boldsymbol{y})\\
A&=f^{gba}\Pi^b_{il}\delta(\boldsymbol{x}-\boldsymbol{y})\\
A'&=-f^{abg}\Pi^b_{li}\delta(\boldsymbol{x}-\boldsymbol{y})\\
B&=2f^{gbc}\Pi^b_{li}(\partial^y_l\delta^{ca}+f^{cde}A^d_l\delta^{ea})\delta(\boldsymbol{x}-\boldsymbol{y})\\
B'&=-2f^{abc}\Pi^b_{il}(\partial^x_i\delta^{cg}+f^{cde}A^d_i\delta^{eg})\delta(\boldsymbol{x}-\boldsymbol{y})\\
U&=\tfrac{1}{4}f^{gbc}F^b_{mn}f^{cae}(F^e_{lm}\delta_{ni}+F^e_{mn}\delta_{li}+F^e_{nl}\delta_{mi})\delta(\boldsymbol{x}-\boldsymbol{y})\\
U'&=-\tfrac{1}{4}f^{abc}F^b_{jk}f^{cge}(F^e_{ij}\delta_{kl}+F^e_{jk}\delta_{il}+F^e_{ki}\delta_{jl})\delta(\boldsymbol{x}-\boldsymbol{y})\\
V&=(\partial^y_l\delta_{il}\delta^{ag}+f^{gba}A^b_i)\delta(\boldsymbol{x}-\boldsymbol{y})\\
V'&=-(\partial^x_i\delta_{il}\delta^{ag}-f^{abg}A^b_l)\delta(\boldsymbol{x}-\boldsymbol{y})\\
W&=\tfrac{1}{2}\delta^{ag}\delta(\boldsymbol{x}-\boldsymbol{y})\\
W'&=-\tfrac{1}{2}\delta^{ag}\delta(\boldsymbol{x}-\boldsymbol{y}).
\end{aligned}
$$

After some calculation, we find the following zero mode solution for equation (39)

$$
\begin{aligned}
\alpha_{ij}&=(D_i\mu_j-D_j\mu_i)+2[B_{ij},\nu]+2[F_{ij},\xi]+2(D_i[\nu,K_j]-D_j[\nu,K_i])\\
\beta_{ij}&=2[\Pi_{ij},\nu]\\
\gamma_i&=2D_i\nu\\
\rho_i&=2[\Pi_i,\nu]\\
\sigma_i&=\mu_i+2D_i\xi\\
\psi_i&=0\\
\phi&=0.
\end{aligned}
\tag{40}
$$

The self-consistency conditions for equations of motion, equation (4),

$$
\Omega^J\equiv(v^J_i)^T\frac{\partial V(q)}{\partial q_i}=0
$$

now vanish identically after replacing the above obtained zero modes:

$$\Omega^{(1)} = \int \mathrm{d}^3\boldsymbol{x} \left\{ \alpha^g_{lm}(\boldsymbol{x}) \frac{\delta}{\delta B^g_{lm}(\boldsymbol{x})} + \beta^g_{lm}(\boldsymbol{x}) \frac{\delta}{\delta \Pi^g_{lm}(\boldsymbol{x})} + \gamma^g_l(\boldsymbol{x}) \frac{\delta}{\delta A^g_l(\boldsymbol{x})} + \rho^g_l(\boldsymbol{x}) \frac{\delta}{\delta \Pi^g_l(\boldsymbol{x})} + \sigma^g_l(\boldsymbol{x}) \frac{\delta}{\delta K^g_l(\boldsymbol{x})} \right\} \int \mathrm{d}^3\boldsymbol{y}\, V_{(1)} \equiv 0.$$

Thus there are no further constraints, and the theory has gauge symmetry whose symmetry transformations are given by the above zero modes. Since γ_i and α_{ij} in equation (40) represent the variations of A_i and B_{ij} under the gauge transformation, respectively, we now see that both the vector and antisymmetric tensor fields have non-Abelian gauge symmetry with their respective gauge parameters ν and μ_i.

Now, the gauge fixing will remove the singularity completely, and we choose the following gauge

$$\partial_i A_i = 0 \qquad D_i B_{ij} = 0 \tag{41}$$

then the Lagrangian becomes

$$\begin{aligned}\mathcal{L} = {} & \tfrac{1}{2}\Pi^a_{ij}\dot{B}^a_{ij} + \tfrac{1}{2}([A_i, K_j] - [A_j, K_i])^a\dot{\Pi}^a_{ij} + [A_j, \Pi_{ij}]^a\dot{K}^a_i + \tfrac{1}{2}\Pi^a_i\dot{A}^a_i + (D_j\Pi_{ji})^a\dot{\eta}^a_i \\ & + (D_i\Pi_i + [B_{ij}, \Pi_{ij}] - 2[\Pi_{ij}, D_jK_i])^a\dot{\omega}^a + [F_{ij}, \Pi_{ij}]^a\dot{\theta}^a \\ & + ([\Pi_{ij}, \Pi_j] - \tfrac{1}{4}[H_{ijk}, F_{jk}])^a\dot{\chi}^a_i + (D_i\eta^a_i + \tfrac{1}{2}\theta^a)\dot{\lambda}^a \\ & + (\partial_i A^a_i)\dot{\zeta}^a + (D_jB^a_{ij})\dot{\tau}^a_i - V_{(2)}\end{aligned} \tag{42}$$

where

$$V_{(2)} = V_{(1)}\,|_{\{\partial_i A_i = 0, D_i B_{ij} = 0\}}\,.$$

Note that here we did not fix the gauge for the auxiliary vector field K, although it behaves like a gauge field with the parameter ξ. This is because the zero mode equation (39) shows that the parameters ξ and μ_i are the variations of θ and η_i, respectively, and θ and η_i are constrained by the reducibility condition (36). Thus the gauge fixing of B_{ij} does the necessary job related to the parameter ξ. One can also check that the symplectic matrix obtained from the above Lagrangian is no longer singular.

4. Discussion and conclusion

In this paper, we analyse the symmetry of the topological mass-generating action in the non-Abelian case with and without a vector auxiliary field. In the Abelian case, the action which does not include a vector auxiliary field develops an effective mass for the vector gauge field when the topological coupling $B \wedge F$ term is present [1]. However, in the non-Abelian case, a straightforwardly extended action of the Abelian type does not provide a gauge symmetry for the antisymmetric tensor field unless one introduces a vector auxiliary field in a specific form. And, if the antisymmetric tensor field does not possess a gauge symmetry, then the physical degree of freedom of the antisymmetric tensor field cannot transmute into a component of the vector gauge field, thus no massive vector gauge field. Hence, it is necessary that both the vector and antisymmetric tensor fields behave as gauge fields.

Recently, it was shown [2, 3] that if a vector auxiliary field is introduced to the action in a specific combination, then both the vector and antisymmetric tensor fields behave as gauge fields. Although the action with full non-Abelian gauge symmetry was constructed and quantized in the BRST formalism in these works [2, 3], the symmetry structure related to the constraints of the theory was not understood completely. In this paper, we fill this gap in the Faddeev–Jackiw formalism.

In section 2, we showed that the vector field transforms as a gauge field, but the antisymmetric tensor field does not, when there is no vector auxiliary field:

$$\delta A_i = 4D_i\nu \qquad \delta B_{ij} = 4[B_{ij}, \nu] \qquad \text{etc.}$$

When we added a vector auxiliary field in section 3, both the vector and antisymmetric tensor fields behaved as gauge fields, that is, both transformations contain derivative terms:

$$\begin{aligned}
\delta B_{ij} &= (D_i\mu_j - D_j\mu_i) + 2[B_{ij}, \nu] + 2[F_{ij}, \xi] + 2(D_i[\nu, K_j] - D_j[\nu, K_i]) \\
\delta A_i &= 2D_i\nu \\
\delta K_i &= \mu_i + 2D_i\xi \\
&\vdots
\end{aligned}$$

In [2], the transformations of fields were given by

$$\begin{aligned}
\delta B_{\alpha\beta} &= D_{[\alpha}\mu_{\beta]} + [B_{\alpha\beta}, \nu] + [F_{\alpha\beta}, \xi] \\
\delta A_\alpha &= D_\alpha\nu \\
\delta K_\alpha &= \mu_\alpha + D_\alpha\xi + [\nu, K_\alpha] \\
&\vdots
\end{aligned} \tag{43}$$

where $\alpha, \beta = 0, 1, 2, 3$. The two transformation laws look apparently different for the antisymmetric tensor and vector auxiliary fields. However, in the action that we adopted in section 3, equation (29), the B-field always appears in the combination of $B_{\alpha\beta} - D_{[\alpha}K_{\beta]}$, and the transformations of this combined field are the same under both transformation rules:

$$\delta(B_{\alpha\beta} - D_{[\alpha}K_{\beta]}) = [B_{\alpha\beta} - D_{[\alpha}K_{\beta]}, \nu].$$

Therefore, the action has the same invariance property under both transformation rules. Notice that should the combined field behave as a covariant scalar, then the auxiliary field K must behave like a gauge field. This symmetry property was the origin of an extra scalar ghost κ in [2]. In general, the antisymmetric tensor of rank 2 or higher must be augmented in such a way that the augmented ones behave like the ordinary two-form field strength under gauge transformation, if antisymmetric tensors are to behave as higher form gauge fields [16]. The above combination of the tensor field and the auxiliary vector field works.

Finally, we turn to the issue of the reducible constraints that appeared in section 3 when the vector auxiliary field was introduced: two primary constraints are related to each other by equation (35). In order to treat these dependent constraints as independent ones, we introduced another constraint expressing this fact. Namely, we added this condition as an additional constraint, equation (36). However, we did not use the relationship between the primary constraints equation (35) as a new constraint. Instead, we used a relationship in which the original primary constraints were replaced by their Lagrange multiplier fields. This is due to the fact that here we imposed the time derivative of a given constraint as a consistency condition instead of the constraint itself. Thus, to impose the relationship among constraints we have to impose the constraint among their multiplier fields. That is what we used in equation (36). This additional condition resolved the reducibility in our case, and we obtained the non-singular symplectic matrix even with a usual gauge choice in equation (41). The reducibility condition also accounts for the apparent lack of gauge fixing for the K field, since this condition also expresses that the gauge parameter ξ is related to the gauge parameter μ_i of the field B_{ij} as we explained in the previous section.

In conclusion, introducing a vector auxiliary field enhanced the symmetry of the action and made both the vector and antisymmetric tensor fields behave as gauge fields. The reducibility of the gauge symmetry of the theory was resolved by introducing a new constraint which properly expresses the relationship among dependent constraints in terms of their Lagrange multiplier fields.

Acknowledgments

We would like to thank Youngjai Kiem for helpful discussions. This work was supported in part by the Ministry of Education, BSRI-96-2442, and KOSEF grant 971-0201-007-2.

References

[1] Allen T J, Bowick M J and Lahiri A 1991 *Mod. Phys. Lett.* A **6** 559
See also Cremmer E and Scherk J 1974 *Nucl. Phys.* B **72** 117
[2] Hwang D S and Lee C Y 1997 *J. Math. Phys.* **38** 30
[3] Lahiri A 1997 *Phys. Rev.* D **55** 5045
Barcelos-Neto J, Cabo A and Silva M B D 1996 *Z. Phys.* C **72** 345
[4] Faddeev L and Jackiw R 1988 *Phys. Rev. Lett.* **60** 1692
[5] Jackiw R 1994 *Constraint Theory and Quantization Methods* ed F Colomo *et al* (Singapore: World Scientific) p 163
[6] Costa M E V and Girotti H O 1988 *Phys. Rev. Lett.* **60** 1771
[7] Barcelos-Neto J and Wotzasek C 1992a *Mod. Phys. Lett.* A **7** 1737
Barcelos-Neto J and Wotzasek C 1992b *Int. J. Mod. Phys.* A **7** 4981
[8] Motani H and Wotzasek C 1993 *Mod. Phys. Lett.* A **8** 3387
[9] Motani H 1993 *Int. J. Mod. Phys.* A **8** 4319
[10] Barcelos-Neto J and Braga N R F 1994 *J. Math. Phys.* **35** 3497
[11] Henneaux M and Teitelboim C 1992 *Quantization of gauge systems* (Princeton, NJ: Princeton University Press) and references therein
[12] Lahiri A 1993 *Mod. Phys. Lett.* A **8** 2403
[13] Barcelos-Neto J and Silva M B D 1995 *Int. J. Mod. Phys.* A **10** 3759
[14] Freedman D Z and Townsend P K 1981 *Nucl. Phys.* B **177** 282
[15] Thierry-Mieg J and Baulieu L 1983 *Nucl. Phys.* B **228** 259
[16] Thierry-Mieg J and Ne'eman Y 1982 *Proc. Natl. Acad. Sci. USA* **79** 7068
[17] Lee C Y and Ne'eman Y 1991 *Phys. Lett.* B **264** 389
Lee C Y and Ne'eman Y 1991 *Phys. Lett.* B **269** 477 (erratum)